Reducing the Passenger Travel Time in Practice by the Automated Construction of a Robust Railway Timetable

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1. Introduction

Automatically generating cyclic timetables has been an active research area for some time (Serafini and Ukovich, 1989; Schrijver, 1993; Odijk, 1996; Nachtigall, 1996; Goverde, 1999; Peeters, 2003; Kroon et al., 2007; Liebchen, 2007), but the application of this research in practice has been limited. We believe this is due to two reasons. Firstly, the current models do not guarantee a feasible solution. Secondly, the resulting solution often contains some very high time supplements because they do not influence the goal function directly. The reason is that these goal functions do often not completely correspond to the true goal of a timetable.

We solve both problems by introduction of the goal function *total passenger time, expected in practice*. Dewilde et al. (2011) give an overview of robustness definitions and conclude that this function is the best criterium for optimisation of a timetable. Since this function evaluates and indirectly steers *all* time related decision variables in the system, we do not need to further restrict the ranges of any of these variables. This allows us to choose larger, more *natural* ranges for them and makes that our model is *always feasible*. Furthermore, some measures are taken to significantly speed up the computation time of our model. These combined features result in our model being *solved more quickly* than previous models. We demonstrate our claims by optimising, in about one hour only, the timetable of all 203 hourly passenger trains in Belgium.

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Assuming an expected primary delay of 2% on the minima of each action during peak hours, the optimised timetable reduces expected passenger time by 7.47%.

2. Optimality: Model Goal Function

Our approach to optimise a timetable for passengers was first described in Sels et al. (2011). It consists of the two steps we call *reflowing* and *retiming*. In the reflowing step we determine the number of passengers in each train on each part of the network (Sels et al., 2011). This information is used in the goal function for the retiming step. The goal function is the total expected passenger time in practice as defined and analytically derived by Sels et al. (2013).

3. Feasibility & Computation Speed: Model Mandatory and Optional Constraints

Our full paper derives the mandatory constraints for our MILP model as given in Table 1 and the optional, solver speed improving constraints in Table 2. Thanks to the natural, large ranges of all decision variables, our model is always feasible. The notation used is defined in Table 4.

Table 1: Mandatory constraints, enforced to generate valid timetables. The notation used is described in Table 4.

intra-process constraints:	$\forall e \in E : b_e + m_e + s_e = e_e$
inter-process	$\forall_{o(e_0)=i(e_1)}(e_0, e_1) \in (E', E' \cup E'') : e_{e_0} = b_{e_1}$
connection constraints:	$\forall_{o(e_0)=i(e_1)}(e_0, e_1) \in (E'', E') : e_{e_0} + d_{e_0} \cdot T = b_{e_1}$
continuous variable bounds:	$\forall_e \in E : 0 \leq s_e \leq T - max(m_e, \delta).$
integer variable bounds:	$\forall e \in E'' : (h_{lo} - h_{hi}) \leq d_e \leq (h_{hi} - h_{lo})$
all trains start in 1^{st} *hour:*	$\forall_{\neg\exists i_d(e)} e \in E_r : h_{lo} \cdot T \leq b_e < (h_{lo} + 1) \cdot T$
passing supplements are 0*:*	$\forall_{\neg s(e)} e \in E_d : s_e = 0$
separate same	$cd(h_l; r_u, r_d, h_r) = h_l \prec \overline{h_l} : \exists r_u \in E_r : r_u = o_r(o(h_l)) :$
direction trains	$\exists r_d \in E_r : r_d = o_r(i(h_l)) : \exists h_r \in E_{hw} : h_r = e_{hw}(o(r_d), o(r_u)).$
on the same open	$\forall_{cd(h_l;r_u,r_d,h_r)} h_l \in E_{hw} : ((m_{h_l} + s_{h_l} + d_{h_l} \cdot T) + (m_{r_u} + s_{r_u})$
track section:	$= (m_{h_r} + s_{h_r} + d_{h_r} \cdot T) + (m_{r_d} + s_{r_d})) \wedge (d_{h_l} = d_{h_r})$
separate opposite	$cd(h_l; r_u, r_d, h_r) = h_l \prec \overline{h_l} : \exists r_u \in E_r : r_u = o_r(o(h_l)) :$
direction trains	$\exists r_d \in E_r : r_d = o_r(i(h_l)) : \exists h_r \in E_{hw} : h_r = e_{hw}(o(r_d), o(r_u)).$
on the same single	$\forall_{cd(h_l;r_u,r_d,h_r)} h_l \in E_{hw} : ((m_{h_l} + s_{h_l} + d_{h_l} \cdot T) + (m_{r_u} + s_{r_u})$
track section:	$+(m_{h_r} + s_{h_r} + d_{h_r} \cdot T) + (m_{r_d} + s_{r_d}) = 0) \wedge (0 \leq d_{h_l} + d_{h_r} \leq 3)$
forbid or allow	$cd(h_l; d_u, d_d, h_r) = h_l \prec \overline{h_l} : \exists d_u \in E_d : d_u = o_d(o(h_l)) :$
overtaking	$\exists d_d \in E_d : d_d = o_d(i(h_l)) : \exists h_r \in E_{hw} : h_r = e_{hw}(o(d_d), o(d_u)).$
within a station	$\forall_{cd(h_l;d_u,d_d,h_r)} h \in E_{hw} : ((m_{h_l} + s_{h_l} + d_{h_l} \cdot T) + (m_{d_u} + s_{d_u})$
depending on	$= (m_{h_r} + s_{h_r} + d_{h_r} \cdot T) + (m_{d_d} + s_{d_d}))$
infrastructure:	$\wedge\ (if\ (\neg iao(d_u, d_d)) : -0 \leq d_{h_l} - d_{h_r} \leq +0,$
& halting patterns:	$else\ if\ (\neg s(d_u) \wedge \neg s(d_d)) : -0 \leq d_{h_l} - d_{h_r} \leq +0,$
	$else\ if\ (s(d_u) \wedge \neg s(d_d)) : -1 \leq d_{h_l} - d_{h_r} \leq +0,$
	$else\ if\ (\neg s(d_u) \wedge s(d_d)) : -0 \leq d_{h_l} - d_{h_r} \leq +1,$
	$else\ if\ (s(d_u) \wedge s(d_d)) : -1 \leq d_{h_l} - d_{h_r} \leq +1)$

Table 2: Optional constraints, only enforced to lower solver time. The notation used is described in Table 4.

opposite transfers induced small cycles (hourglasses):	$\forall_{t<\bar{t}} t \in E_{tr} : ((m_t + s_t + d_t \cdot T) + (m_{\bar{t}} + s_{\bar{t}} + d_{\bar{t}} \cdot T)$ $= (m_{d_u} + s_{d_u}) + (m_{d_d} + s_{d_d})) \wedge (-1 \leq d_t + d_{\bar{t}} \leq 0)$
opposite dwell-begin-headway induced small cycles (forward triangles):	$cd(h; d_u, t_u, d_d, t_d) = h \prec \bar{h} : \exists d_u \in E_d : d_u = o_d(h) :$ $\exists t_u \in E_{tr} : t_u = e_{tr}(i(h), o(d_u)) :$ $\exists d_d \in E_d : d_d = o_d(\bar{h}) : \exists t_d \in E_{tr} : t_d = e_{tr}(o(h), o(d_d)).$ $\forall_{cd(h;d_u,t_u,d_d,t_d)} h \in E_{hw} : ((m_h + s_h + d_h \cdot T) + (m_{d_u} + s_{d_u})$ $= (m_{t_u} + s_{t_u} + d_{t_u} \cdot T)) \wedge (0 \leq d_{t_u} - d_h \leq 1)$ $\wedge((m_h + s_h + d_h \cdot T) + (m_{t_d} + s_{t_d} + d_{t_d} \cdot T) = (m_{d_d} + s_{d_d}))$ $\wedge(-1 \leq d_{t_d} + d_h \leq 0)$
opposite dwell-end-headway induced small cycles (backward triangles):	$cd(h; d_u, t_u, d_d, t_d) = h \prec \bar{h} : \exists d_u \in E_d : d_u = i_d(\bar{h}) :$ $\exists t_u \in E_{tr} : t_u = e_{tr}(i(d_u), i(h)) :$ $\exists d_d \in E_d : d_d = i_d(h) : \exists t_d \in E_{tr} : t_d = e_{tr}(i(d_d), o(h)).$ $\forall_{cd(h;d_u,t_u,d_d,t_d)} h \in E_{hw} :$ $((m_h + s_h + d_h \cdot T) + (m_{t_u} + s_{t_u} + d_{t_u} \cdot T)$ $= (m_{d_u} + s_{d_u})) \wedge (-1 \leq d_{t_u} + d_h \leq 0)$ $\wedge((m_h + s_h + d_h \cdot T) + (m_{d_d} + s_{d_d})$ $= (m_{t_d} + s_{t_d} + d_{t_d} \cdot T)) \wedge (0 \leq d_{t_d} - d_h \leq 1)$
opposite headway integer constraints:	$\forall_{h<\bar{h}} h \in E_{hw} : ((m_h + s_h + d_h \cdot T)$ $= -(m_{\bar{h}} + s_{\bar{h}} + d_{\bar{h}} \cdot T)) \wedge (d_h + d_{\bar{h}} = -1)$
transfer induced Dijkstra cycle constraints:	$\forall t \in E_{tr} : \sum_{e\in(c_t^+ \cap E')} m_e + s_e + \sum_{e\in(c_t^+ \cap E'')} m_e + s_e + d_e \cdot T$ $= \sum_{e\in(c_t^- \cap E')} m_e + s_e + \sum_{e\in(c_t^- \cap E'')} m_e + s_e + d_e \cdot T$

4. Results

We constructed a model with all constraints shown in Tables 1 and 2 and the objective function derived in Sels et al. (2013), for all 203 hourly passenger train relations in Belgium departing between 7 and 8am in the timetable of 13 March 2013 timetable. Table 3 shows the results of our optimisations, which assume different primary delay distributions, controlled by the parameter a. Table 3 also shows, for the case $a = 2\%$, that the optimised timetable further improves when the MILP gap decreases, at the expense of more solver time. Compared to the original timetable, our optimised timetables have quite some advantages. First, they respect all minimum ride- and dwell-times. Second, they respect all headway time buffers of 3 minutes between all train pairs on the same track section. Third, our calculations show that, (for $a = 2\%$), the average chance of missing a transfer in the current timetable is 14.1% while in our optimised timetable (for $gap = 75.9\%$), it is only 2.3%. The expected passenger time of this optimised schedule is 7.47% lower than in the original schedule. Fourth, generating our schedule only takes about one hour, while it takes many human planners many months to generate the current timetable.

Table 3: Increasing primary delays, characterised by their average of $a\%$ of minimum dwell and ride times. The first column shows $a\%$. Column 2 and 3 show the computation time and the MILP gap achieved. We ran Gurobi 5.5.0 on an Apple MacBook Pro with a 2.6GHz Intel i7 processor and 16GB 1600MHz DDR3 memory. For the first set of result rows, the gap desired was set slightly above what was obtained as the gap of the first returned solution in earlier trials. The results in the last row are obtained by reduction of the desired gap by 1% compared to the first row. Graph size: 203 hourly trains, 5355 ride, 5152 dwell, 17553 major transfer, 31696 knock-on(=headway) and 166 turn-around edges. Model size: 42609 b and 42609 s decision variables. 42609 e expressions. 49415 d decision variables. 41128 goal function terms for major flows.

a	solver time	MILP gap	major flows linearised time reduction	major flows non-linearised time reduction	all flows linearised time reduction	all flows non-linearised time reduction	missed transfer probability original	missed transfer probability optimised
%	min.	%	%	%	%	%	%	%
2	53	75.9	9.15	7.47	3.39	1.68	14.1	2.3
4	63	70.7	6.92	5.03	1.33	-0.58	14.6	3.1
6	39	63.9	6.44	4.24	0.47	-1.76	15.1	1.8
8	66	61.3	4.49	2.29	-0.93	-3.16	15.6	4.4
2	114	74.8	9.79	7.95	3.62	1.71	14.1	2.5

5. Conclusions

Our full paper has the following main contributions. Firstly, our MILP model, unlike others, is always feasible, so our system always returns a solution. This assumes that the number of trains being scheduled do not exceed the available capacity. Naturally, any of our generated timetables respects all minimal ride, dwell, transfer, headway and turn-around time rules. Secondly, our goal function results in timetables with minimised expected passenger time, meaning the total passenger journey time, including their ride, dwell and transfer actions, as well as the typical primary delays and their consequential knock-on delays in practice are minimised. This means our generated timetable is also both efficient and robust by construction. Thirdly, this timetable is also quickly generated, in about one hour only. Fourthly, supposing primary delay distributions with an average of 2% of minimum times of each ride and dwell action, our improved timetable reduced expected passenger time for realistic passenger streams by 7.47%.

Finally, while in MILP modelling, restricting search space and using simplified goal functions are the easier measures often taken to reduce solver times, we show that defining an all-encompassing goal function and searching the full solution space can lead to more desirable results: guaranteed feasibility, optimality and low solver times, even for hard problems.

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Table 4: Notation used. Note that d_e is defined $\forall e \in E''$, while b_e, m_e, s_e, e_e are defined $\forall e \in E' \cup E''$.

T	= cyclic timetable period	δ	= timetable time resolution	
h_{lo}	= first hour of schedule	h_{hi}	= last hour of schedule	
E'	= set of primary edges	r	= ride	b_e = begin time of e
E'	$= E_r \cup E_d$	d	= dwell	m_e = minimum time of e
E''	= set of secondary edges	hw	= headway	s_e = supplement time of e
E''	$= E_{tr} \cup E_{hw} \cup E_{ta}$	ta	= turn-around	e_e = end time of e
E	= set of all edges	tr	= transfer	d_e = integer variable for e
E	$= E' \cup E''$	t	= edge type	V = vertex set
$i(e)$	= in vertex of e	$i_t(v)$	= unique type t inbound edge of vertex v	
$o(e)$	= out vertex of e	$o_t(v)$	= unique type t outbound edge of vertex v	
$e_t(v_0, v_1)$	= unique type t edge from vertex v_0 to vertex v_1	$i_t(e)$	= unique type t predecessor edge of edge e	
		$o_t(e)$	= unique type t successor edge of edge e	
$s(d)$	= true iff dwell action d is a stop	$iao(d_0, d_1)$	= true iff station infrastructure allows overtaking between trains with dwell actions d_0 and d_1	
$\bar{e}$ is the edge between trains t_B and t_A, that is the *opposite* of the edge e from train t_A to t_B				
cd functions set the conditions under which rules apply and also define names for edges				

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