# Towards a Better Train Timetable for Denmark <br> Reducing Total Expected Passenger Time 

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#### Abstract

With our Periodic Event Scheduling Problem (PESP) based timetabling method, including the objective function of total expected passenger journey time in practice, we are able to produce a passenger robust timetable for all 88 hourly passenger trains in Denmark. This timetable reduces the expected journey time of all Denmarks train passengers together by $2.9 \%$ compared to the timetable defined by BaneDanmark as the timetable for a general Wednesday in 2014. The computation of our timetable takes only 65 minutes.


Keywords Expected Passenger Time • Integer Linear Programming • Optimal Cyclic Timetabling • Periodic Event Scheduling Problem

Mathematics Subject Classification (2000) 90C11;90C06

## 1 Introduction

We previously constructed a Periodic Event Scheduling Problem (PESP) based model which has as objective function: the total expected passenger journey time in practice over all passengers together [15]. In [2], the authors conclude that, unlike to what is the case for some alternative definitions of robustness,

[^0]this objective function is a practical method to obtain robustness and that the obtained robustness is ideal for passengers.

In [15], Sels et al. applied this MIP model to the set of all 196 hourly trains in Belgium. The main results were that a timetable, automatically generated in about 2 hours, saves about $3.8 \%$ of total expected passenger journey time. This timetable also significantly reduced the percentage of missed transfers from $13.9 \%$ to $2.6 \%$. To study how generally applicable this model is to practice, we now also test it on the set of all 88 hourly trains in Denmark.

## 2 Methodology and Assumptions

Our timetabling approach consists of the basic constraints of the popular PESP model $[16,11,9,3,4,10,5,7,6,1,17]$ over its standard event activity network. We impose its classic constraints enforcing minimal ride times and minimal dwell times. As described in detail in [12], we automatically construct all potential transfers. By this, we mean that if two trains stop in the same station, a transfer edge will be added between the arrival time of the feeder train and the departure time of the target train. Currently, a minimum of 3 minutes is assumed for each transfer. Headway edges and constraints are also automatically constructed between entry times of each pair of trains that enter an infrastructure resource and similarly also between all pairs of exit times. For single track sections, between each leaving and each entering train, a similar headway time constraint is imposed. The headway minimum time assumed on this macroscopic level is 3 minutes. This summarises all hard constraints in our model.

As described in detail in [13] and [14], our objective function consists of the sum of the expected passenger time for each event activity network edge that corresponds to a passenger action. So, for each ride, dwell and transfer edge there is an expected passenger time. We express this expected passenger time of an edge as a function of its minimum time and its added supplement time. The shape of this function mainly depends on the expected primary delay distribution and consequently, so does the supplement that should be ideally added. The scale of this function depends on the number of passengers involved. This indicates the relative importance of the expected passenger time of one edge compared to that of another and is balanced by the objective function. For the primary delays, as do $[8,3,18]$, we assume negative exponential distributions with an average that can be set to a certain fixed percentage $a$ of the minimum time for that action. For now, we assume the same value of $a$ for all ride, dwell and transfer edges, for all trains and for all locations. The value of $a$ is typically chosen in the range of $1 \%$ to $5 \%$ [3].

For ride and dwell actions, the expected time, as a function of the added ride and dwell supplement $s$, is almost the identity function $f(s)=s$. This is logical, since for whatever supplement is added to a ride or dwell action, the ride or dwell passengers just have to sit it through. So high values of s are not beneficial to these passengers. At low values of $s$ the slope of $f(s)$ is a little flatter because increasing a small supplement $s$ by some supplementary seconds
has more of a buffering effect against delays than increasing a supplement that is already higher by the same amount. This is the case because small delays occur more frequently than large delays.

For transfer actions, we model an expected transfer time that depends on the chosen supplement for this transfer, on top of the minimum of 3 minutes. If the supplement is low, the probability that the transfer is missed is high. If the transfer is missed, we conservatively assume a penalty waiting time of the timetable period, here 1 hour. If the supplement is high, the probability of missing the transfer is low, but the transfer passenger will always have to wait for a time equal to the supplement. The above means that the expected passenger time for a transfer is a U-shaped function of the supplement. So there is a trade-off and a locally optimal value for the transfer supplement.

As for secondary delays, or knock-on delays, our model already contains the graph edges associated to these. Indeed, they are the same edges as the headway edges, temporally separating pairs of trains that use the same infrastructure resource. So for each headway edge, we also add a term in the objective function that represents the knock-on time or secondary delay that passengers on the second train may experience in case the first train is delayed. In our model, as derived in [14], this time depends on the delay distributions of both trains and on the number of passengers on the second train. Obviously, the total knock-on time is proportional to the number of passengers on the second train. Also, the expected knock-on passenger time forms a decreasing function of the train separating supplement $s$, since the higher the separation, between two trains, the lower the expected knock-on delay.

This concludes our discussion of all expected passenger time components that passengers will experience in reality. The objective function of our model is the sum of all these. When minimising this function, a timetable with minimised total expected passenger time will result. Note that, often, there will be competition between the terms in the objective function. For example, selecting a locally well balanced supplement on a transfer taken by a first group of people may be ideal for them, but may cause another group of people dwelling for somewhat longer on the target train of this transfer. Since all objective function terms are expressed in units of people times time, the total objective function fairly and properly balances costs and benefits of every supplement choice in the network.

## 3 Results

For this project, Banedanmark started from the Danish timetable for an average Wednesday in 2014 and slightly adapted it so that it became exactly periodical with one hour. This timetable contains 84 passenger trains and 4 freight trains. On the event activity network resulting from these trains, we applied our passenger routing algorithm as described in [12] in order to obtain the passenger number for every edge in the graph. Note that the freight trains in our system start in a technical station that passengers do not have access to. The freight trains also do not stop in passenger stations and so, in our routing
algorithm, no passengers can get on or off these trains, as is the case in practice. Next we perform timetabling, according to the methodology described in section 2, using the obtained local passenger numbers as fixed weights in the objective function. Each of our timetabling models was tackled by the MILP solver Gurobi version 6.0 .0 on an Intel Xeon E31240 3.3GHz processor with 16 GB of RAM. The results are shown in table 1.

Table 1 Results for different timetable optimisations of all 88 hourly Danish trains. req. $=$ required, obt. $=$ obtained, exp. time $=$ expected passenger time, red. $=$ reduction, eval. $=$ evaluation, orig. $\mathrm{tt}=$ original timetable, opt. $\mathrm{tt}=$ optimised timetable, $\mathrm{rd} .+\mathrm{dw} . \mathrm{t}=$ ride + dwell train time.

| $\begin{gathered} 1 \\ \mathrm{a} \\ (\%) \end{gathered}$ | $\begin{gathered} 2 \\ \text { transfer } \\ \text { threshold } \end{gathered}$ | 3 <br> gap <br> req. <br> (\%) | 4 <br> gap <br> obt. <br> (\%) | 5 solver time (s) | 6 exp. time red.eval. (\%) | 7 <br> missed <br> orig.tt <br> (\%) | 8 ansfers: opt.tt (\%) | $\begin{aligned} & 9 \\ & \text { planned } \\ & \text { rd.+dw. } \mathrm{t} \\ & \text { red.(\%) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 420 | 75 | 74.92 | 19421 | 1.67 | 11.34 | 2.07 | -4.88 |
| 2 | 420 | 74 | 73.63 | 62417 | 1.96 | 11.34 | 5.21 | -3.95 |
| 2 | 210 | 79 | 78.07 | 1534 | 0.82 | 11.34 | 2.83 | -7.77 |
| 2 | 210 | 77 | 76.62 | 2436 | 1.59 | 11.34 | 3.20 | -4.89 |
| 2 | 210 | 75 | 74.96 | 2924 | 2.45 | 11.34 | 1.12 | -3.08 |
| 2 | 210 | 74 | 73.83 | 3922 | 2.90 | 11.34 | 2.45 | -2.53 |
| 2 | 210 | 73 | 72.96 | 20726 | 3.16 | 11.34 | 2.07 | -2.05 |
| 2 | 195 | 76 | $\geq 76.8$ | $\geq 101000$ |  |  |  |  |

Results for the different optimisations and their respective input parameter values are ordered from less to more demanding from top to bottom. By more demanding, we mean that either the required MIP gap (column 3) is lower or the number of transfers considered in the optimisation is higher or a combination of both. The transfer threshold (column 2) is the number of people that are required as minimum for a transfer to be considered in the optimisation.

We see that setting the transfer threshold to 420 makes that the solver spends a lot of time (19421 and 62417 seconds) before it finds a solution with an optimality gap below the required one. When the transfer threshold is lowered to 210 transfer passengers, resulting in more transfers considered in the optimisation, the model seems to become easier for Gurobi. When subsequently also lowering the required gap from $79 \%$ to $74 \%$ (column 3), timetable solutions are found within 1534 to 3922 seconds (column 5) and corresponding savings of total expected passenger time increase from $0.82 \%$ to $2.90 \%$ (column 6 ). Lowering the required gap further to $73 \%$ still improves the solution with a total reduction of expected passenger time of $3.16 \%$, however, the computation time then increases significantly to 20726 seconds, being 5.76 hours.

So, we investigate whether, with a required gap of $76 \%$, a better solution could be obtained by lowering the transfer threshold. The last line of table 1
shows that after 101000 seconds, no acceptable timetable solution was found yet, since the solver is still at a gap of $76.8 \%$.

Table 1 also mentions that the expected missed transfer probability is $11.34 \%$ (column 7) for the current timetable while not more than $2.45 \%$ $(1.12 \%, 2.45 \%$ and $2.07 \%$, column 8) for our best three timetables. These results were obtained by a post optimisation calculation on the obtained timetables, where expected delays are accumulated and resulting in fractions of missed and non-missed transfers.

For the best timetables found, the last column of table 1 mentions that these possess between $3.08 \%$ and $2.05 \%$ more train weighted planned ride and dwell time than the original timetable. Even then, the total passenger time is reduced. This is possible due to a number of factors. Firstly, our method adds supplements to trains but weighs them by passengers. Secondly, supplements can cause extra robustness, so adding planned time can reduce experienced time in practice. Thirdly, classical manual timetabling uses rules of thumb like assigning a certain percentage of supplement to each train. To avoid knockon delays, we expect these rules to perform worse than our rule of assigning supplements between each couple of trains sharing an infrastructure resource, even more so since we do this proportionally with the number of passengers on the second train and the expected delay distributions of both trains.

The resulting timetable was verified by Banedanmark by visual inspection of space-time graphs per infrastructure line. No train collisions nor violations of minimal headway times were found. Further verification of realistic parameter settings like the value of $a$ and the value of transfer minima is warranted for fair comparison with the current timetable. Also verification of other timetable quality criteria like the possible preference to avoid large inserted supplements, even for actions with very few passengers, is required and ongoing.

## 4 Conclusion

This paper demonstrates that our PESP based method with the extension of an objective function representing total expected passenger time in practice improves the timetable for the whole train network of Danish passenger trains. Total passenger time is reduced by $2.9 \%$ and transfers become significantly more reliable. The fact that, after our successful application to the Belgian train network, the application to a second country now delivers satisfying results as well, indicates that our approach is quite generally useful.

Since the computation time for this timetable is only 65 minutes, this could lead to huge time savings in the current timetabling practice which, for the biggest part, is still carried out manually. Alternatively, the time spent on manual timetabling now, can instead be used to create more alternative line planning proposals which can be fed to our timetabling system. The line plan leading to the optimised timetable with the lowest total expected passenger time can then be selected. This would further improve passenger service.

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