Temporal Spreading of Alternative Trains in order to Minimise Passenger Travel Time in Practice

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Business Problem

Belgian Infrastructure Management Company: Infrabel:

Find Timetable that Minimises Expected Passenger Travel Time (includes: ride, dwell, transfer time and primary & secondary delays)

Note:

Reduce Expected Passenger Time $\Rightarrow$ Optimises Robustness

Fixed:

Infrastructure, Train Lines, Halting Pattern, Primary Delay Distributions

Variable:

Timing: Supplement Times at every Ride, Dwell, Transfer Action,
$\Rightarrow$ variable inter-Train Heading Times $\Rightarrow$ variable Train Orders

Specifics:

One Busy Day, Morning Peak Hour

Context: FAPESP: Two Phased

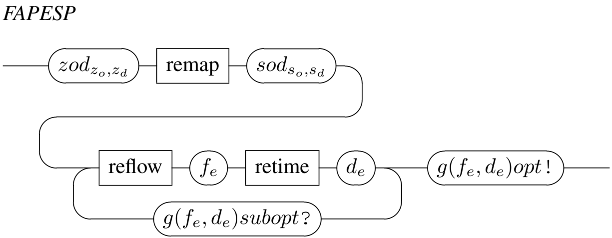
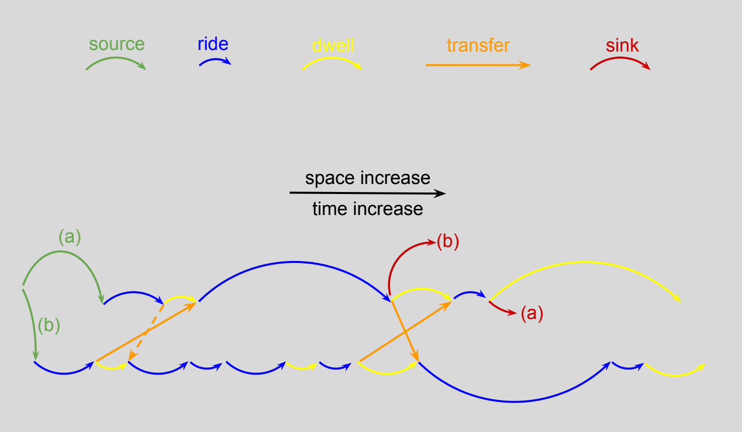
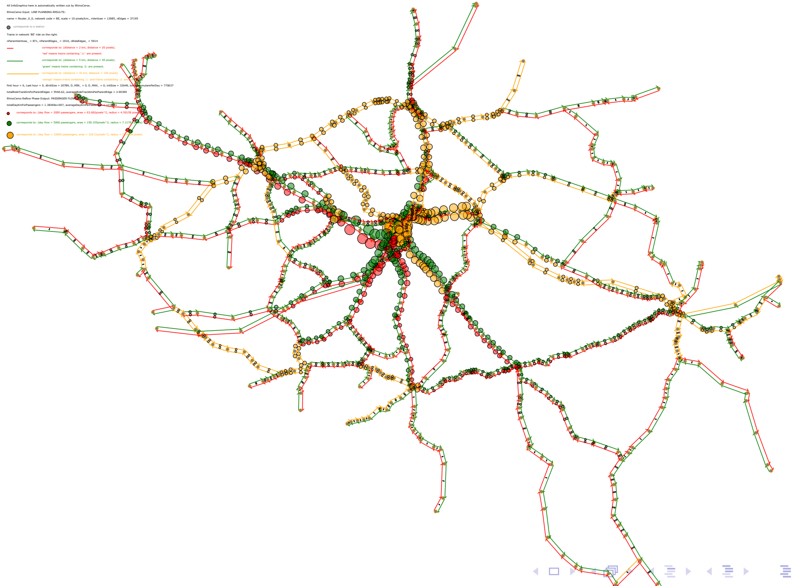


Figure: Two Phased implies Iterations

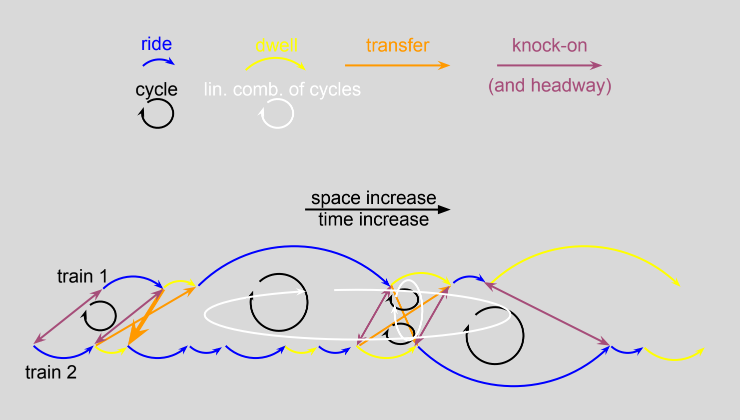
Graph for Reflowing: add Source & Sink Edges



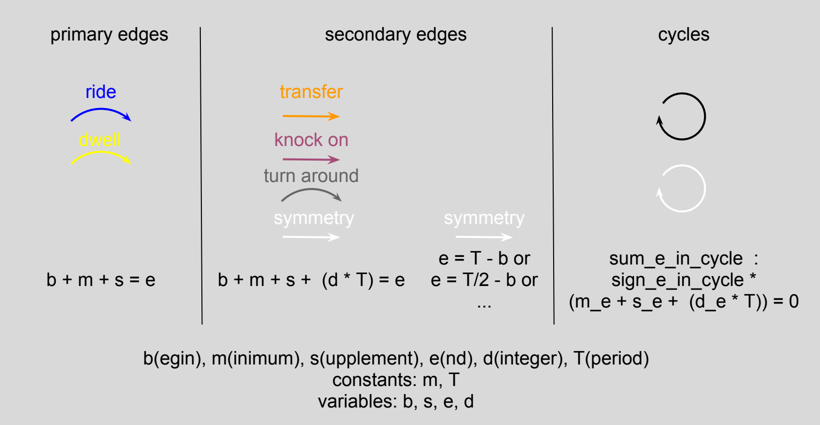
Result of Reflowing: Disc Area = Daily Flow



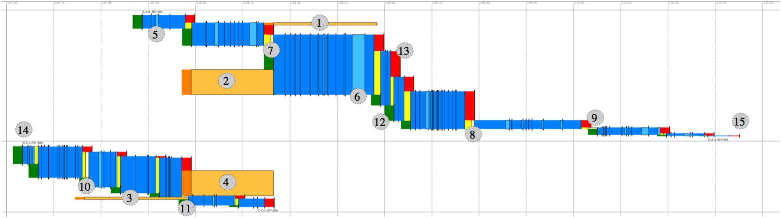
Graph for Retiming: add Knock-On Edges & Cycles



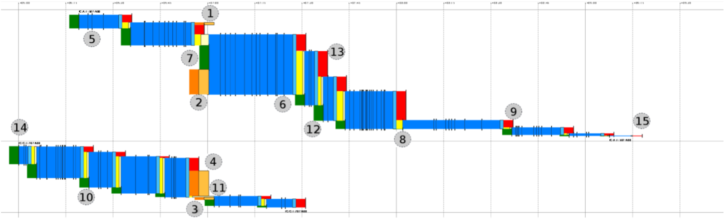
Graph for Retiming: All Constraints



Reflowing decides on Rectangle Heights
Retiming (Timetabling) decides on Rectangle Widths

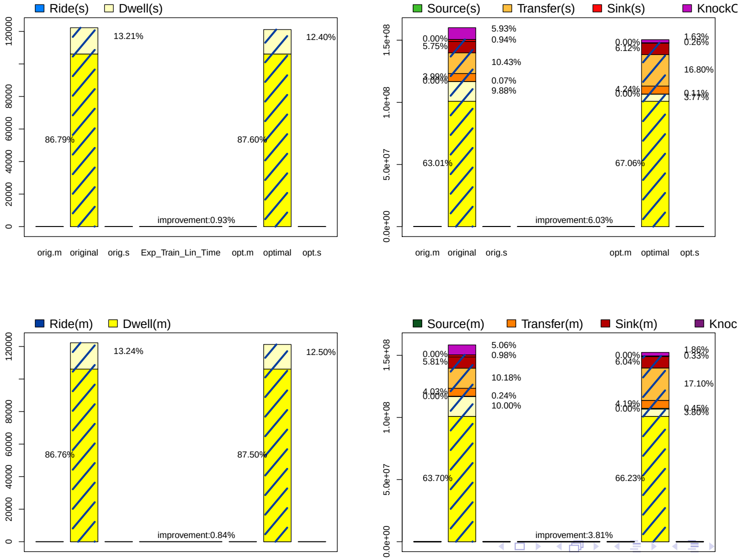


(a) Original Schedule



(b) Optimized Version

Expected (Non-)Linear Time, as used in Evaluation



Cui Bono? To Whose Benefit?

"It is a Latin adage that is used either to suggest a hidden motive or to indicate that the party responsible for something may not be who it appears at first to be."

Cui Bono? To Whose Benefit?

- Passengers arriving at station randomly minimise their waiting time before departure
- "inter-departure waiting time"
- "inter-arrival waiting time"
- do benefit:
  - random arrival passengers (fraction r)
- do NOT/barely benefit:
  - passengers adapting to train departure schedule (fraction $1 - r$)

Categories Determining r

<table>
<tr><td rowspan="6">informed</td><td rowspan="5">caring</td><td rowspan="4">adaptable</td><td rowspan="4">adapting</td><td rowspan="2">in time</td><td>for dep.</td><td rowspan="2">$(1 - r)$</td><td rowspan="2">passenger choice model</td></tr>
<tr><td>for trsfr.</td></tr>
<tr><td rowspan="2">over time</td><td>for dep.</td><td rowspan="5">r</td><td rowspan="5">random arrival model</td></tr>
<tr><td>for trsfr.</td></tr>
<tr><td>unadaptable</td><td rowspan="3" colspan="3">not adapting</td></tr>
<tr><td colspan="2">not-caring</td></tr>
<tr><td colspan="3">uninformed</td></tr>
</table>

Table: Sub-categories of passengers: fraction (1-r) showing passenger choice model behaviour and fraction r showing random arrival behaviour. (dep. = departure, trsf. = transfer)

How Much do they Benefit/Suffer?

[Welding(1957)], [Holroyd and Scraggs(1966)], [Osuna and Newell(1972)]:

$$E(w) = E(h)/2 \cdot \left(1 + C_v(h)^2\right) \quad (1)$$

$$C_v(h) = \sigma(h)/\mu(h) \quad (2)$$

$$E(h) = \sum_{i=0}^{N-1} p_i \cdot H_i = \sum_{i=0}^{N-1} (H_i/T) \cdot H_i = \sum_{i=0}^{N-1} H_i^2/T \quad (3)$$

$$E(f \cdot w) = \frac{f}{2T} \sum_{i=0}^{N-1} H_i^2 \cdot \left(1 + C_v(h)^2\right) \quad (4)$$

ILP Model: Constraints

$$\forall (O,D):\begin{cases} \forall i \in I_N \setminus \{N-1\}: & \overline{b}_i \leq \overline{b}_{i+1} \\ & \overline{b}_{N-1} \geq \overline{b}_0 \end{cases} \quad (5)$$

$$\forall (O,D): \forall i \in I_N: \begin{cases} \overline{b}_i = \sum_{j \in I_N} p_{i,j} \cdot b_j \\ \forall j \in I_N : p_{i,j} \in \{0,1\} \\ \sum_{j \in I_N} p_{i,j} = 1 = \sum_{j \in I_N} p_{j,i} \end{cases} \quad (6)$$

$$\forall (O,D): \begin{cases} \forall i \in I_N \setminus \{N-1\} & : & s_i & = & \overline{b}_{i+1} - \overline{b}_i \\ & & s_{N-1} & = & (\overline{b}_0 + T) - \overline{b}_{N-1} \\ \forall i \in I_N & : & 0 & \leq & s_i \leq T - \delta \end{cases} \quad (7)$$

$\forall (i,j) \in I_n : 0 \leq b_i \leq T - \delta$ and so for the non-decreasingly ordered $\overline{b}_i$ it holds that $\forall i \in I_N \setminus \{N-1\} : 0 \leq \overline{b}_{i+1} - \overline{b}_i \leq T - \delta$.

$$\forall (O,D): \sum_{i \in I_N} s_i = T \quad (8)$$

ILP Model: Objective Function = How Much Penalty?

Passengers arriving at O and wanting to go to D, before having taken first train:

- their expected waiting time for randomly arriving passengers between train i and train i+1 is

$$u_i = \int_0^{s_i} (s_i - t) \cdot f \, dt = s_i f \int_0^{s_i} dt - f \int_0^{s_i} t \, dt = f \frac{s_i^2}{2}. \qquad (9)$$

- their total expected waiting time for randomly arriving passengers is

$$\forall (O, D) : U = \sum_{i \in I_N} u_i = f/2 \cdot \sum_{i \in I_N} s_i^2. \qquad (10)$$

ILP Model: Piecewise Linearisation

$$\forall (O,D) : \forall i \in I_N : \begin{cases} (s_{i,0}, u_{i,0}) = (0,0) \\ (s_{i,1}, u_{i,1}) = \left(\frac{T}{N}, \frac{f}{2}\left(\frac{T}{N}\right)^2\right) \\ (s_{i,2}, u_{i,2}) = \left(T, \frac{f}{2} T^2\right) \end{cases} \tag{11}$$

$$\forall (O,D) : \forall i \in I_N : \begin{cases} u_i \geq u_{i,0} + \frac{u_{i,1} - u_{i,0}}{s_{i,1} - s_{i,0}} \cdot (s_i - s_{i,0}) \\ \quad = 0 + \frac{\frac{fT^2}{2N^2}}{\frac{T}{N}} (s_i - 0) = \frac{fT}{2N} s_i \\ u_i \geq u_{i,1} + \frac{u_{i,2} - u_{i,1}}{s_{i,2} - s_{i,1}} \cdot (s_i - s_{i,1}) \\ \quad = \frac{fT^2}{2N^2} + \frac{\frac{fT^2}{2} - \frac{fT^2}{2N^2}}{T - \frac{T}{N}} \left(s_i - \frac{T}{N}\right) \\ \quad = \frac{fT^2}{2N^2} + \frac{N}{(N-1) \cdot T} \left(\frac{fT^2 N^2 - fT^2}{2N^2}\right) \left(s_i - \frac{T}{N}\right) \\ \quad = \frac{fT^2}{2N^2} \left[1 + \frac{N(N+1)}{T} \left(s_i - \frac{T}{N}\right)\right] \end{cases} \tag{12}$$

ILP Model: Variable Linearisation

Introduce helper variables $h_{i,j}$, such that

$$\forall (O,D) : \forall i,j \in I_N : h_{i,j} = p_{i,j} \cdot b_j, \tag{13}$$

which is in linearised form:

$$\forall (O,D) : \forall i,j \in I_N : \begin{cases} (bl - bu)(1 - p_{i,j}) & \leq h_{i,j} - b_j & \leq (bu - bl)(1 - p_{i,j}) \\ bl \cdot p_{i,j} & \leq h_{i,j} & \leq bu \cdot p_{i,j}. \end{cases} \tag{14}$$

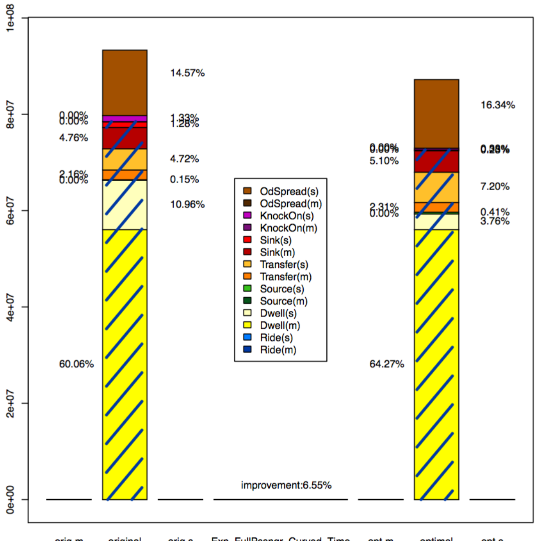
So, now

$$\forall (O,D) : \forall i \in I_N : \overline{b}_i = \sum_{j \in I_N} p_{i,j} \cdot b_j \tag{15}$$

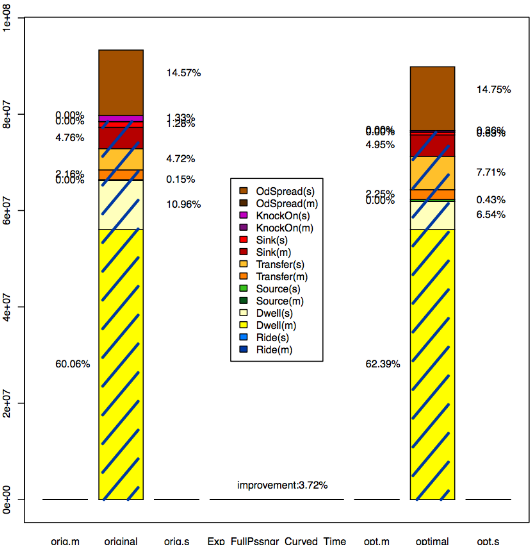
can be replaced with:

$$\forall (O,D) : \forall i \in I_N : \overline{b}_i = \sum_{j \in I_N} h_{i,j}. \tag{16}$$

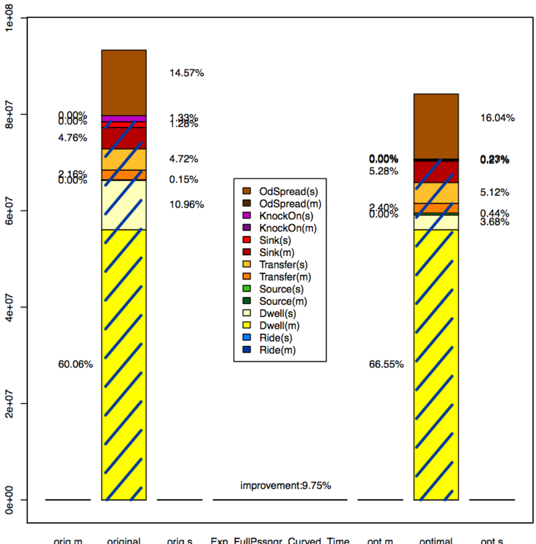
Results Graphical, 26 trains, (Soft, Soft) (O,D)-Spreading



Results Graphical, 26 trains, (Hard, Hard) (O,D)-Spreading



Results Graphical, 26 trains, (Hard, Soft) (O,D)-Spreading



Results

- 26 trains
  - with transfers
    - soft-soft spreading, 6.55% reduction
    - hard-hard spreading: 3.72% reduction: saves most spreading time, but overstretched, so more ride & dwell time
    - hard-soft spreading: 9.75% reduction
- 200 trains
  - no solution yet

Conclusions

- derived and implemented cost function that measures inter-departure wait time:
    - to evaluate and optimise a schedule on total Excess Journey Time
    - which is addable to other expected passenger time (ride, dwell, transfer, knock-on times)
    - that cannot render the model infeasible
- currently still computationally challenging

Future Work

- try to control computation time by:
  - manual i.o. automatic selection of corridors that need spreading
- try to decrease computation time by:
  - addition of spreading specific cycles
  - fixing some alternative train orders (breaks 'symmetry', since they are 'the same')
- compare (computation time and solution quality) with classical method of imposing temporal spreading via hard constraints

Questions / Next Steps

- Your questions?
  - here and now, or ...
  - sels.peter@gmail.com
  - www.LogicallyYours.com/research/
- My questions:
  - percentage of non-adapting passenger $r = ?\%$
  - tips/tricks to reduce computation time?

- Holroyd, E. M., Scraggs, D. A., 1966. Waiting Times for Buses in Central London. Traffic Engineering Control. 8, 158–160.
- Osuna, E. E., Newell, G. F., 1972. Control Strategies for an Idealized Public Transportation System. Transportation Science. 6 (1), 57–72.
- Welding, P. I., 1957. The Instability of Close Interval Service. Operational Research Society. 8 (3), 133–148.